The Effect of Multipath Propagation on the Performance of DPIM on Diffuse Optical Wireless Communications

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\textbf{Abstract}

This paper investigates the performance of digital pulse interval modulation (DPIM) in the presence of multipath propagation and additive white Gaussian noise. The results for optical power requirements normalised to the average optical power required by the on-and-off keying (OOK) verses the channel normalised delay spread for DPIM without guard band and with guard bands are presented, and are compared with PPM and OOK. Results show that for a given order, DPIM with no guard band has a lower bandwidth requirement than PPM and OOK, and consequently has a lower ISI power penalty than PPM employing threshold detection. By introducing a guard band, the performance of DPIM in the presence of ISI can be improved considerably.

\textbf{1. Introduction}

In both diffuse and nondirected line of sight (LOS) infrared link configurations, the transmitted optical signal may undergo multiple reflections from the walls, ceiling, floor and objects within a room, before arriving at the surface of the photodetector. In the case of the diffuse configuration, where a LOS path does not exist, all the transmitted signal incident on the photodetector surface has experienced at least one reflection. This multipath propagation causes the transmitted pulses to spread in time, resulting in intersymbol interference (ISI). The effect of ISI is significant for bit rates above 10 Mbit/s [1].

In this paper, the performance of DPIM is analysed in the presence of multipath propagation and additive white Gaussian noise (AWGN), which is independent of the received signal. Interference from artificial sources of ambient light is ignored. Throughout the paper, all optical power requirements are normalised to the average optical power required by OOK, operating at a given bit rate, to achieve a PER of $10^{-6}$ on an ideal channel, i.e. with an impulse response $\delta(t)$, limited only by AWGN. Packet lengths are assumed to be 1 kbyte.

The remainder of this paper is organised as follows: In section 2, the parameters used to quantify the severity of the ISI are defined, and channel model used in the analysis is described. The unequalised performance of DPIM is evaluated in section 3. One method of improving the performance of DPIM in the presence of ISI involves placing a guard band in each symbol immediately following the pulse. In section 3 the effectiveness of this technique is analysed, and the results are compared with those obtained for DPIM without a guard band. The findings of the paper are discussed in section 4, and a conclusion is given in section 5.

\textbf{2. Optical Channel Model}

The multipath channel, described by its impulse response $h(t)$, is fixed for a given position of the transmitter, receiver and intervening reflectors, and changes significantly only when any of these are moved by distances in the order of centimetres [2]. Due to the high bit rates under consideration and the relatively slow movement of people and objects within a room, the channel will vary significantly only...
on the time scale of many bit periods, and hence, it is justifiable to model the channel as time invariant. The power penalties associated with the channel may be separated into two factors: optical path loss and multipath dispersion [3,4]. The optical gain for a channel with impulse response \( h(t) \) is defined as:

\[
G_o = \int_{-\infty}^{\infty} h(t) \, dt, \tag{1}
\]

and the average received optical signal power \( P_{rx} = G_o P_{tx} \), where \( P_{rx} \) is the average transmitted optical signal power. Hence, optical path loss (dB) = \(-10 \log_{10} G_o\).

In this paper, consideration is limited to the power penalty due to multipath propagation only. Consequently, the channel impulse response is divided by \( G_o \) in order to give unity area. In this case \( P_{rx} = P_{tx} \). Since the channel \( h(t)/G_o \) has the same unity gain as the ideal channel \( \delta(t) \), only the effect of multipath dispersion is measured. The RMS delay spread \( D_{RMS} \) is a parameter, which is commonly used to quantify the time dispersive properties of multipath channels, and is defined as the square root of the second central moment of the magnitude squared of the channel impulse response [5], [6]. \( D_{RMS} \) can be found from the channel impulse response using the following [7]:

\[
D_{RMS} = \sqrt{\frac{\int_{-\infty}^{\infty}(t - \mu)^2 h^2(t) \, dt}{\int_{-\infty}^{\infty} h^2(t) \, dt}}, \tag{3}
\]

where \( \mu \) is the mean delay, given by:

\[
\mu = \frac{\int_{-\infty}^{\infty} t h^2(t) \, dt}{\int_{-\infty}^{\infty} h^2(t) \, dt}. \tag{4}
\]

Practical channel measurements by Kahn et. al. [7] have shown that both nondirected LOS and diffuse configurations have channel RMS delay spreads which typically range form 1 to 12 ns, with diffuse links generally being slightly larger. As one may expect, shadowing is more detrimental to nondirected LOS channels, increasing delay spreads to typically between 7 and 13 ns, whilst on diffuse channels, the increase in delay spread due to shadowing is relatively modest. It has been shown that there is a systematic relationship between multipath power penalty and normalised delay spread \( D_T \), which is a dimensionless parameter defined as the channel RMS delay spread (3) divided by the bit duration [7]. This relationship implies that a single parameter model is sufficient to calculate ISI power penalties on all four types of nondirected channel. To predict ISI power penalty the model used is based on the ceiling bounce model [3], [4], which is given by:

\[
h(t, a) = \frac{6a^6}{(t + a)^7} u(t), \tag{5}
\]

where \( u(t) \) is the unit step function and \( a \) is related to the RMS delay spread by:

\[
D_{RMS}(h(t, a)) = \frac{a}{12} \sqrt{\frac{13}{11}}. \tag{6}
\]

For OOK and PPM schemes, both unequalised and equalised operation, the model was found to predict multipath power requirements on nondirected LOS and diffuse channels, with and without shadowing, with a high degree of accuracy. However, no multipath analysis based on the above model for DPIM has been reported yet. In this paper, for the first time, we investigate the performance of DPIM with the multipath channel is modelled using the ceiling bounce model (5), normalised such that \( G_o = 1 \).
3. Unequalised Performance
3.1 PIM with no guard band

In this section, the performance of DPIM is analysed without a guard band present. In order to avoid any confusion later, this will be referred to as DPIM(NGB). A block diagram of the unequalised DPIM(NGB) system under consideration is shown in Fig. 1. The input bits, assumed to be independent, identically distributed (i.i.d) and uniform on \{0, 1\}. The encoder maps each block of \(\log_2 L\) input bits to one of \(L\) possible DPIM(NGB) symbols, see Fig. 4. The symbols are passed to a transmitter filter, which has a unit-amplitude rectangular impulse response \(p(t)\) with a duration of one slot \(T_s\) given as: [8]

\[
T_s = T_b \log_2 L / L_{\text{avg}}
\]

where \(L_{\text{avg}} = (L + 1)/2\) is the mean symbol length in slots. Such a slot duration gives the same average bit rate as OOK / PPM, assuming that all symbols are equally likely. The output of the transmitter filter is scaled by the peak transmitted optical signal power \(P_{\text{avg}}\), and passed through the multipath channel \(h(t)\). The received optical signal power is converted into a photocurrent by multiplying by the photodetector responsivity \(R\). AWGN \(n(t)\) is added to the detected signal, which is then passed to a unit energy filter with an impulse response \(r(t)\), which is matched to \(p(t)\). The filter output is sampled at the slot rate, and a threshold detector then assigns a one or zero to each slot depending on whether the sampled signal is above or below the threshold level. The BER may be calculated using the method proposed in [1], which is described as follow. Let \(c_k\) donate the discrete-time equivalent impulse response of the cascaded system, which is given by:

\[
c_k = P(t) \otimes h(t) \otimes r(t) \bigg|_{t = kT_s}.
\]

Unless the channel is non-dispersive, \(c_k\) contains a zero tap, a single precursor tap and possibly multiple postcursor taps. The magnitude of the zero tap is larger than the magnitudes of the other taps. On a non-dispersive channel, the optimum sampling point, i.e. that which minimises the probability of error, occurs at the end of each bit period. However, on dispersive channels, the optimum sampling point changes as the severity of ISI changes. In order to isolate the power penalty due to ISI, two assumptions are made. Firstly, perfect timing recovery is assumed. This is achieved by shifting the time origin so as to maximise the zero tap \(c_0\) [1]. Secondly, optimal decision threshold is assumed. The optimum sampling point is selected by maximising the zero tap. Assuming \(c_k\) has \(m\) taps, all possible \(m\)-slot PIM sequences are generated, and their corresponding occurrence probabilities are calculated. Note that, since the slots are not i.i.d., different \(m\)-slot sequences may have different occurrence probabilities, and when \(m > 2\), the total number of valid sequences is always less than \(2^m\). For any \(m\)-slot DPIM(NGB) sequence, denoted as \(\mathbf{b}_i\), the input to the threshold detector for the \((m-1)\)th slot period, in the absence of noise, is given as:

\[
y_i = L_{\text{avg}} R P_{\text{avg}} \mathbf{b}_i \otimes c_k \bigg|_{k = m}
\]

Since slots containing a one are less likely than empty slots, the optimum threshold level does not simply lie midway between expected one and zero values, as it does for OOK. It is a complicated function of the signal and noise powers, the discrete-time impulse response \(c_k\), the order \(L\), and the severity of the ISI, and is determined iteratively in the analysis. The probability of bit error for the \((m-1)\)th slot of sequence \(\mathbf{b}_i\) is given as:

\[
\varepsilon_j = \begin{cases} 
Q \left( \frac{y_i - \alpha}{\sqrt{N_0 / 2}} \right) & \text{if } b_i = 1 \\
Q \left( \frac{\alpha - y_i}{\sqrt{N_0 / 2}} \right) & \text{if } b_i = 0
\end{cases}
\]

\[(10)\]
The average probability of slot error $P_{\text{slot}}$ is found by multiplying the probability of slot error for the $(m-1)^{\text{th}}$ slot of a given $m$-slot sequence by the probability of occurrence for that particular sequence, and summing over all possible sequences. Therefore,

$$P_{\text{slot}} = \sum_{i=1}^{L} p_{E_{i}}$$  \hspace{1cm} (11)

Note that unlike PPM, there are $2^m$ possible $m$-slot sequences for all $m$, though the occurrence probabilities are different since the slots are not i.i.d. Using the approximation that the number of slots contained within a $D$-bit packet is $\sim L_{\text{avg}} D / \log_2 L$, $P_{\text{slot}}$ may be converted into a corresponding PER using:

$$\text{PER} = 1 - (1 - P_{\text{slot}})^{\frac{L_{\text{avg}} D}{\log_2 L}}$$  \hspace{1cm} (12)

Note that the actual bit duration used is irrelevant in this analysis, since the RMS delay spread is normalised to it. The parameter $a$ is determined using (6), and from this, the channel impulse response is calculated using (5). The discrete-time impulse response of the cascaded system is then determined, with the sampling instants chosen so as to maximise $c_0$. The average transmitted optical signal power is then varied until the target PER of $10^{-6}$ is achieved. Fig. 2 shows a plot of average optical power requirement versus the normalised delay spread for various orders of DPIM(NGB). Also shown for comparison is the plot for OOK and PPM, employing threshold detection. As shown in Fig. 2a, the power requirement increases approximately exponentially as $D_T$ increases. Beyond $D_T \sim 0.52$ the error rate becomes irreducible, i.e. the target PER cannot be achieved simply by increasing the transmit power. However, for PIM and PPM schemes the power requirements for higher order cases increase more rapidly than the lower order cases. This is due to the lower slot duration, which means that a greater number of slots are affected by the ISI. When the ISI is severe enough, $L = 4$ actually offers a lower power requirement than $L = 32$ for both DPIM and PPM, see Fig. 2a & b. For low values of $D_T$, DPIM(NGB) requires slightly higher power and lower power lower compared with the PPM and OOK, respectively due to the increased power efficiency. On an ideal channel these difference is $\sim 0.8$ and 0.5 dB, respectively. However, as $D_T$ increases, the power requirement curves intersect, and DPIM(NGB) then offers a lower power requirement. For example, for $D_T = 0.3$ and $M = 8$, DPIM(NGB) requires 1.3 dB less power compared with the PPM(TH) and 1.3 more power compared with the OOK. Both PIM(NGB) and PPM(TH) have irreducible error rates which occur at very similar values of $D_T$.

If the $x$- and $y$- axes of Fig. 2a are changed to represent the ratio of RMS delay spread to slot duration and optical power penalty due to ISI, respectively, it can be observed that the relationship between these two quantities is almost the same for all orders of DPIM(NGB), as shown in Fig. 3. The same phenomenon has also been observed for PPM [9] and DPPM [10, 11]. For all values of $D_T$ DPIM(NGB) has a lower ISI power penalty than PPM(TH).

### 3.2 DPIM with guard band

Intuitively, when a DPIM slot sequence is passed through a multipath channel, the postcursor ISI is most severe in the slots immediately following a pulse. Using this fact, the unique symbol structure of DPIM may be exploited to provide a simple method of improving error performance in the presence of ISI. This method involves placing a guard band, which consists of one or more guard slots, in each symbol immediately following the pulse. Upon detection of a pulse, the following slot(s) contained within the guard band are automatically assigned as zeros, regardless of whether or not the sampled output of the receiver filter is above or below the threshold level. Thus, the postcursor ISI present in these slot(s) has no effect in system performance, provided that the pulse initiating the symbol is correctly detected. The mapping of source data to transmitted symbols for 4DPIM with no guard band (NGB), with a guard band consisting of one slot (1GS) and with a guard band consisting of 2 guard slots (2GS) is shown in Fig. 4. The shaded areas in the figure represent the guard slots. The inclusion of a guard band increases the average number of slots per symbol, and consequently, in order to maintain the same average bit rate, it is necessary to reduce the slot duration. For DPIM(1GS) and DPIM(2GS) the slot duration is given in
(7), where \( L_{avg} = (L + 3)/2 \) and \((L+5)/2\), respectively. On its own, a reduction in slot duration would result in increased optical power requirements, since the ISI would affect a greater number of slots. Therefore, in order for the guard band to achieve a net reduction in optical power requirement, the reduction in power due to the presence of the guard band must outweigh the increase in power due to the reduced slot duration.

For DPIM(1GS), the probability of error for any given slot is dependent not only on the sampled signal value corresponding to that particular slot, but also on the sampled value of the previous slot. Thus, when evaluating DPIM(1GS), if \( c_b \) has \( m \) taps, it is necessary to generate sequences of length \((m+1)\), and evaluate the probability of slot error for the \( m^{th} \) slot, using sample values for the \( m^{th} \) and \((m-1)^{th}\) time slots. Note that by including a guard band, for a sequence length of \((m+1)\) slots, not all the \(2^{m+1}\) possible DPIM(NGB) sequences are actually valid, since the guard band excludes all sequences which contain adjacent pulses. The guard band does, however, increase the maximum run length of consecutive zeros.

In order to explain the function of the guard band, the following notation is used. Let \( b_m \) and \( b_{m-1} \) represent the values of the \( m^{th} \) and \((m-1)^{th}\) slots in a DPIM(1GS) sequence, respectively, where \( b_m, b_{m-1} \in \{0,1\} \). Let \( \hat{b}_m \) and \( \hat{b}_{m-1} \) represent the estimate of \( b_m \) and \( b_{m-1} \), respectively, after passing through the multipath channel. Since the probability of slot error for the \( m^{th} \) slot in a sequence is affected by the detection of the \((m-1)^{th}\) slot, there are 4 possible detection scenarios:

i) if \( b_{m-1} = 1 \) and is correctly detected, \( b_m \) must be a zero and \( \hat{b}_m \) is automatically assigned a zero. Therefore, there is no chance of an error in slot \( m \). ii) if \( b_{m-1} = 1 \) but is falsely detected as a zero, \( b_m \) must be a zero but \( \hat{b}_m \) is not automatically assigned a zero. Therefore, an error will occur if \( \hat{b}_m = 1 \). iii) if \( b_{m-1} = 0 \) and is correctly detected, \( b_m \) could be either a one or zero, and the \((m-1)^{th}\) slot has no effect on the probability of error in slot \( m \). iv) if \( b_{m-1} = 0 \) but is falsely detected as a one, \( \hat{b}_m \) is automatically assigned a zero, but an error will occur if \( \hat{b}_m = 1 \).

The probability of slot error for DPIM(1GS) cannot easily be expressed in a concise form. Consequently, pseudo code is used. In conjunction with the above notation, let \( y_m \) and \( y_{m-1} \) be the sampled output of the receiver filter corresponding to the \( m^{th} \) and \((m-1)^{th}\) slots, and \( \alpha \) be the optimum threshold level. For any given \((m+1)\)-slot DPIM(1GS) sequence, the probability of slot error in the \( m^{th} \) slot may be calculated as follows:

\[
\begin{align*}
\text{if } b_{m-1} = 1 & \& \hat{b}_{m-1} = 1 \quad \{ b_m = 0 \text{ and } \hat{b}_m \text{ is automatically set to } 0 \} \\
P_{\text{slot}} = 0
\end{align*}
\]

\[
\begin{align*}
\text{elseif } b_{m-1} = 1 & \& \hat{b}_{m-1} = 0 \quad \{ b_m = 0 \text{ but } \hat{b}_m \text{ is not automatically set to } 0 \} \\
P_{\text{slot}} = Q((\alpha - y_m) / \sqrt{N_0/2})
\end{align*}
\]

\[
\begin{align*}
\text{elseif } b_{m-1} = 0 & \& \hat{b}_{m-1} = 0 \quad \{ b_m \text{ could be } 1 \text{ or } 0 \} \\
\text{if } b_m = 1 \\
P_{\text{slot}} = Q((y_m - \alpha) / \sqrt{N_0/2})
\end{align*}
\]

\[
\begin{align*}
\text{else} \\
P_{\text{slot}} = Q((\alpha - y_m) / \sqrt{N_0/2})
\end{align*}
\]

\[
\begin{align*}
\text{end}
\end{align*}
\]

\[
\begin{align*}
\text{else } b_{m-1} = 0 & \& \hat{b}_{m-1} = 1 \quad \{ \hat{b}_m \text{ is automatically set to } 0, \text{ but } b_m \text{ could be } 1 \} \\
\text{if } b_m = 1 \\
P_{\text{slot}} = 1
\end{align*}
\]

\[
\begin{align*}
\text{else} \\
P_{\text{slot}} = 0
\end{align*}
\]

\[
\begin{align*}
\text{end}
\end{align*}
\]
As before, in order to calculate the average probability of slot error, the above pseudo code is used to calculate the probability of slot error for the $m^{th}$ slot in every possible $(m+1)$ slot sequence. These probabilities are then multiplied by their corresponding occurrence probabilities and the results are summed. The average probability of slot error may then be converted into a corresponding PER using (12), with the substitution $L_{avg} = (L + 3)/2$. For each value of $D_T$ in the range $10^{-3}$ to $0.4$, the optical power requirement was calculated, using the optimum sampling point and threshold level, for various orders of DPIM(1GS) and DPIM(2GS). For DPIM the normalised optical power requirements are plotted in Fig. 5. At low values of $D_T$, adding a guard band reduces the average duty cycle of the transmitted signal and hence, gives a reduction in optical power requirement. This effect is more pronounced at lower orders, where the average duty cycle is reduced by a greater percentage. As $D_T$ increases, the difference between the NGB and 1GS curves increases, thus highlighting the effectiveness of adding a single guard slot. At normalised delay spreads where DPIM(NGB) experiences irreducible error rates, the power requirements are finite for DPIM(1GS). Adding a second guard slot gives a further reduction in power requirements at low values of $D_T$. Again, this is more pronounced at lower orders. For high normalised delay spreads, the improvement in performance over 1GS is clear, with irreducible error rates occurring at higher values of $D_T$ than they do for 1GS. For intermediate normalised delay spreads however, adding a second guard slot results in only a marginal reduction in power requirement. The reason for this can be explained with the aid of Fig. 6, which shows the optical power penalty due to ISI versus normalised delay spread. From Fig. 6 it may be observed that at intermediate values of $D_T$ ($10^{-2} < D_T < 10^{-1}$), 2GS gives a higher power penalty than 1GS, indeed for $L = 32$, there is a region where 2GS has a higher power penalty than NGB. This means that the benefit of adding a second guard slot is outweighed by the reduction in slot duration required to accommodate it. Therefore, in these regions, the power requirements of 2GS converge with those of 1GS. Even with the use of a guard band, OOK still has a lower power requirement at high values of $D_T$, see Fig. 2c. The inclusion of the guard band simply increases the value at which the crossover takes place. For example, considering $L = 32$, OOK offers a lower power requirement compared with NGB for $D_T > 0.13$, whilst this crossover does not take place until $D_T = 0.18$ for 1GS and $D_T = 0.31$ for 2GS.

4. Discussion

In the absence of multipath dispersion, DPIM, just like PPM, can achieve improvements in power efficiency by increasing its order. However, increasing the order also increases the bandwidth requirement, which on multipath channels results in a greater ISI power penalty. When $D_T$ is sufficiently large, the reduction in power requirement due to the reduced duty cycle is outweighed by the increased ISI power penalty, and it becomes more efficient to switch to a lower order. Therefore, for unequalised DPIM, and PPM the most power efficient order on any given channel is a function of the normalised delay spread. For a given order, DPIM(NGB) has a lower bandwidth requirement than PPM, and consequently has a lower ISI power penalty than PPM(TH). However, at low normalised delay spreads, PPM(TH) is more power efficient due to its reduced duty cycle. Consequently, the power requirements of the two schemes intersect at a certain value of $D_T$. Above this value, DPIM(NGB) offers the lower power requirement, though both schemes do give irreducible error rates at very similar values of $D_T$.

By introducing a guard band, the performance of DPIM in the presence of ISI can be improved considerably. In order to achieve a net reduction in optical power requirement, the reduction in power requirement due to the effectiveness of the guardband in mitigating postcursor ISI must outweigh the increase in power requirement due to the increased bandwidth required to accommodate the guard band. On channels where the ISI is nominal, the presence of a guard band gives a reduction in optical power requirement due to the reduced average duty cycle of the transmitted signal. As the severity of ISI increases, the difference in performance between DPIM with and without a guard band increases, thus highlighting the effectiveness of the guard band. Normalised delay spreads which result in irreducible error rates without a guard band give finite power requirements when a guard band is used. At intermediate values of $D_T$, the reduction in power requirement offered by 2GS compared with 1GS is minimal. However, increasing the length of the guard band from one to two slots becomes worthwhile at high values of $D_T$. 
5 Conclusions

In this paper, the performance of DPIM in the presence of multipath propagation and additive white Gaussian noise has been analysed, and compared with the more established techniques of OOK and PPM. For low values of normalised delay spread, PPM has lower power requirement than DPIM (NGB). However, DPIM offers lower ISI power penalty, and consequently as delay spread increases, it offers a lower power requirement. The addition of a guard band further improves the performance of DPIM in the presence of ISI.

6- References


Fig. 1. Block diagram of the unequalised DPIM(NGB) system.
Fig. 2a. PIM (NGB) normalised optical power requirement Vs. normalised delay spread for different values of $L$.

Fig. 2b. PPM normalised optical power requirement Vs. normalised delay spread for different values of $L$ employing threshold detector: $L = 4$, $L = 8$, $L = 16$, $L = 32$.

Fig. 2c. OOK normalised optical power requirement Vs. normalised delay spread.

Fig. 3. DPIM optical power penalty Vs. normalised RMS delay spread.

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<tr>
<th>Source Data</th>
<th>4-DPIM Symbols</th>
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Fig. 4. Mapping of source data to 4-DPIM symbols for NGB, 1GS and 2GS.
Fig. 5. DPIM normalised optical power requirement Vs. normalised delay spread with and without guard bands for (a) $L = 4$, (b) $L = 8$, (c) $L = 16$ and (d) $L = 32$.

Fig. 6. DPIM Optical power penalty Vs. normalised delay spread.