Bit-error rate analysis for hybrid PIM-CDMA optical wireless communication systems

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ABSTRACT: A hybrid pulse interval modulation – code division multiple access (PIM-CDMA) is proposed. Expression for bit error rate (BER) is given. Numerical results presented are compared with hybrid PPM-CDMA and OOK-CDMA schemes. For a low number of users PIM-CDMA achieves similar and marginally lower BER performance compared with PPM-CDMA and OOK-CDMA, respectively. For large number users it offers better performance compared with OOK.

Key words: Pulse modulation; CDMA; PIM; optical wireless; Strict-OOC
1. INTRODUCTION

There have been a growing number of research interests in using optical wireless for future indoor communication systems. It offers larger bandwidth, free from spectrum regulation, ability to re-use the same frequency (or wavelength) within a large room or adjacent rooms, and immunity to radio frequency interference [1-2]. Optical wireless channel, just like optical fibre, has practically unlimited resources which can support image-based and multimedia applications, and multiple access. To implement the latter, there are several multiplexing schemes available, namely, time-division multiple-access (TDMA), wavelength-division multiple-access (WDMA) and code-division multiple-access (CDMA). Compared with the others, CDMA offers numerous advantages, namely, the time-frequency domain is fully utilised with no need for time synchronisation and frequency management between each transmitter and receiver, flexibility in network design, and security against interception. However, CDMA offers less transmission capacity than TDMA [3]. In CDMA, each user is assigned a length of unique signature sequence or address code, which is encoded on each individual user’s information. At the receiver, the information is recovered by correlating the incoming CDMA signature sequences with the same assigned signature sequence as in the transmitter [4]. Therefore, the pattern of each unique signature sequence is crucial since it carries the intended receiver address and helps to avoid the information being overlapped when carrying out correlation at the receiver. Wong et al [5] have shown that use of a signature sequence can mitigates the effect of multi-path dispersion and artificial light interference.

There are two main families of signature sequences, of unipolar (0,1), that has been proposed for optical systems. They are (i) optical orthogonal code (OOC) [4,6,7] and (ii) Prime code (PC)
OOC, which offers unique correlation properties, achieves improved bit-error-rate (BER) performance compared with PC [10]. However, the algorithm used for generating OOC [7] is more complex compared with PC [8]. Both families are designed for use in a constant-bit-rate (CBR) system. Zhang has shown that OOC can be used in a variable-bit-rate (VBR) system, but the drawback is that its correlation properties are violated thus resulting in an increased BER [11,12]. In order to avoid correlation properties violation, guard slots must be inserted between any two successive signature sequences prior to transmission. However, this will result in increased signature length, which in turn results in increased modulation bandwidth, and system complexity. Nevertheless a new OOC family, known as strict-OOC [11,12], was proposed that is suitable for variable-bit/multirate and even constant-bit-rate applications, where the correlation properties are guaranteed preserved. Strict-OOC achieves this without the need for bandwidth expansion and increase in the system complexity, compared with the conventional OOC. Strict-OOC can also be used in optical wireless CDMA system, since no synchronisation is required between the users. In this paper, we use strict-OOC in conjunction with a hybrid CDMA system.

In diffuse optical wireless system PPM-CDMA has been reported, which offers improved power efficiency compared with OOK-CDMA [13,14,15] but lower bit rate enhancement factor (BEF) [15]. A hybrid PPM-CDMA [16,15] proposed is more power efficient offering higher BEF, and efficient utilisation of the modulation bandwidth compared with the OOK-CDMA. Pulse interval modulation (PIM) has also been proposed for optical wireless communication systems due to its bandwidth and power efficiencies and built in slot and frame synchronisation compared with PPM and OOK [17]. In this paper, a hybrid PIM-CDMA employing strict-OOC address code is proposed for wireless network, which offers higher BEF compared with both the hybrid PPM-
CDMA and OOK-CDMA schemes. For a large number of users, it also offers lower bit error rate and marginally higher error rate compared with OOK-CDMA and hybrid PPM-CDMA, respectively. Before introducing hybrid PIM-CDMA and its bit rate analysis, a brief discussion on reported CDMA schemes are presented in the following section.

2. CDMA SCHEMES

OOK-CDMA was the first CDMA scheme proposed for fibre optic communication systems of $N$ users using $(n, w, \lambda_a, \lambda_c)$ OOC as signature sequences [4,6], where $n$ is the sequence length of $w$ weight and, $\lambda_a$ and $\lambda_c$ are the auto- and cross-correlation constraints, respectively. Both $\lambda_a$ and $\lambda_c$ are usually set to one to achieve the optimum BER performance [4]. The signature sequence $C^i$ for the $i^{th}$ user can be represented in binary format as $\left(a^i_1, a^i_2, \ldots, a^i_n\right)$, where $i = 1, 2, \ldots, N$ and $a^i_j \in (0,1)$ is the binary patterns in the $j^{th}$ slot that characterises $C^i$. The number of $a^i_j = 1$ is equal to $w$. Note that $a^i_1$ is always equal to “1” in order to initialise the start of signature sequence. On the other hand, the sequence can be represented in short as a set of codeword given as:

$$C^i = \{j^i_1, j^i_2, \ldots, j^i_n\}, \quad (1)$$

where $j^i_q$ denotes the slot position of the $q^{th}$ binary pattern $a^i_j = 1$. In this system, a signature sequence of length $n$, for the $i^{th}$ transmitter, is transmitted to represent the bit “1”, and where an empty sequence, also of length $n$, to represent the bit “0”. As an example, Fig. 1 shows OOK data stream (1011), and its corresponding OOK-CDMA symbol structure for the first transmitter. The signature sequence $C^1 = \{1,2\}$ is obtained from a (7, 2, 1, 1) strict-OOC family and is used throughout this paper.
Hybrid PPM-CDMA was proposed to enhance the data rate for a given fixed bit rate $R_b$ [16,15]. It maps $M$ bits of information onto a fixed frame of duration $T_f = (2^M + n - 1)T_b$, representing one of $2^M$ distinct symbols. In each symbol, the $a_i^j$ of the first transmitter’s signature sequence is located at a position corresponding to the decimal value $S_k^i$ of $M$ bits data, where $S_k^i \in \{0,1,2,...,2^M - 1\}$ is for the $k^{th}$ symbol. Figure 2b shows an example of two hybrid PPM-CDMA symbols mapping two bits of the information for the first transmitter.

Both OOK-CDMA and hybrid PPM-CDMA schemes have fixed symbol length and thus require frame synchronisation. When $S_k^i$ is less than its maximum value $(2^M - 1)$, hybrid PPM-CDMA frames will have a number of unused (empty) time slot, $x_i^j = ((2^M + n - 1) - (n + S_k^i)T_b$ following the signature sequence carrying no information. Whereas, hybrid pulse interval modulation (PIM)-CDMA scheme is a fully asynchronous system offering self-synchronisation [17] and improvement in the data rate by eliminating the unused time slots $x_i^j$ (see Fig. 2c). In the following section, hybrid PIM-CDMA characteristics, its BEF improvement and its implementation are discussed. Finally, the bit-error rate analysis is presented in section 3.1, followed by discussion and concluding remarks.

3. HYBRID PIM-CDMA

3.1. THEORY

As with $L$-PPM, $L$-PIM maps independent, identically distributed input bits of the $i^{th}$ user into one of $L=2^M$ possible symbols. A symbol is composed of a pulse of one slot of duration, $T_b$, followed by $m$ empty slots, where $0 \leq m \leq L-1$. However, unlike $L$-PPM, symbol duration is
variable and determined by the information content $S_k^i$, which is the value of $m$ for the $k^{th}$ symbol. In order to avoid symbols that have no slots between adjacent pulses (i.e. $m=0$), a guard slot may be added to each symbol immediately following the pulse [17]. However, with hybrid PIM-CDMA scheme employing strict-OOC, the additional guard slot is unnecessary, and this condition has been explained in [18].

Each hybrid PIM-CDMA symbol frame is composed of a signature sequence $C^i = (a_1^i, a_2^i, ..., a_n^i)$ of fixed length $nT_b$ located at the start of the symbol, followed by $S_k^i T_b$ number of empty information slots. Each encoded symbol has variable duration of $(n+S_k^i) T_b$.

Figure 2c shows an encoded hybrid PIM-CDMA symbol frames’ format mapping 2 bits of the information.

Assuming that symbols transmitted by each user is equal likely, then each average hybrid PIM-CDMA data rate is $R_{pin} = 2MR_b/(2n + L - 1)$, and its BEF normalised to OOK and PPM are

$BEF_{pin/ook} = 2nM/(2n + L - 1)$, and $BEF_{pin/ppm} = 2(n + L - 1)/(2n + L - 1)$, respectively. For a given value of $n$ and for $M \leq 8$ the $BEF_{pin/ook} \approx M$ and $BEF_{pin/ppm} \approx 1$. However, for $8 < M < 10$ the $BEF_{pin/ook} < M$, and decreasing rapidly for $M > 10$, whereas, $BEF_{pin/ppm}$ improves by about 15% for $M > 8$. Increasing $n$ will increase further the BEF, by about 10-15%, for $M > 8$. For hybrid PIM-CDMA the optimum $BEF$ is at $2^M = n/2$, see [18].

Suppose that a system is composed of $N$ pairs of transmitter and receiver (i.e. one user) each transmitting independently and asynchronously. Since each pair is identical, therefore only the $i^{th}$
pair is shown in Fig. 3a. The only difference between pairs is the assigned signature sequences, which are obtained from a \((n, w, \lambda_a, \lambda_c)\) S-OOC family. As shown in Fig. 3a, an \(M\) bits data is first converted into a PIM symbol having a variable length of \((n + S^i_k)T_b\). A pulse, \(g(t)\) of duration \(T_b\), is located at the start of each symbol to indicate both the start of the symbol and the first pulse, \(a^i_1\), of the signature sequence. The CDMA encoder, composed of tapped delay lines, then spreads out the rest of \(C^i\), i.e. \(a_2, a_3, \ldots, a_n\), with reference to \(a^i_1\) in each frame. The infinite PIM symbol stream and hybrid PIM-CDMA symbol stream are given by:

\[
d^i_{pim}(t - \tau^i_{pim}) = \sum_{k=\infty}^{\infty} g\left(t - \tau^i_{pim} - \left[1 + nk + \sum_{l=\infty}^{k-1} S^i_l\right]T_b\right),
\]

\[
e^i_{pim}(t - \tau^i_{pim}) = \sum_{k=\infty}^{\infty} \sum_{j=1}^{n} a^i_j g\left(t - \tau^i_{pim} - jT_b - \left[nk + \sum_{l=\infty}^{k-1} S^i_l\right]T_b\right),
\]

where \(l\) denotes the \((k-1)^{th}\) symbol and \(0 \leq \tau^i_{pim} < (2^M + n - 1)T_b\) is the delay.

The receiver block diagram is shown in Fig. 3b, where the received hybrid PIM-CDMA signals, \(\sum_{r=1}^{N} e^r_{pim}\), is first correlated with the same signature sequence \(C^i\) as that of the \(i^{th}\) transmitter. The correlator generates an autocorrelation waveform having a dominant peak equal to the code weight of \(C^i\) when it receives the right sequence (i.e. \(C^i\) in \(e^i\)). However, when it receives different code sequence \(C^y\), where \(y=1, 2, \ldots, N\) and \(y \neq i\), it generates a cross-correlation waveform. The cross-correlation constraint, \(\lambda_c\), is equal to the number of coincidence of 1’s in all time-shifted sequence of the \(C^i\) and \(C^y\). The output at the correlator is given by:
\[ h^i(t) = \sum_{i=1}^{\infty} \sum_{j=1}^{n} a^i_j e_j^{\text{pim}} (t - \tau^i_{\text{pim}} - jT_b) \]  
\[ = w \sum_{k=-\infty}^{\infty} g \left(t - \tau^i_{\text{pim}} - (n-1)T_b - \left[1 + nk + \sum_{j=0}^{k-1} S^i_j \right]T_b \right) + I_{N}^i(t) \]  
\[ = wg(t^i_1) + I_{N}^i(t^i_1) + I_{N}^i(t^i_2) \]

where \( t^i_1 \in \{\text{time slots where a signal pulse occurs}\} \) and, \(-\infty < t^i_2 < \infty \) and \( t^i_2 \neq t^i_1 \). The first term in (4) represents the PIM symbol stream with an offset of \((n-1)T_b\) which is due to correlation process, and the remaining two terms represent the interfering signal due to self-interference and other simultaneous users interference, which is given by:

\[ I_{N}^i(t) = \left[ \sum_{j=1}^{n} \sum_{k=1}^{\infty} a^i_j e_j^{\text{pim}} (t - \tau^i_{\text{pim}} - jT_b) + \sum_{j=1}^{n} \sum_{j=1}^{N} a^i_j e^j_{\text{pim}} (t - \tau^j_{\text{pim}} - jT_b) \right] - wg(t^i_1) \]

The minimum signal amplitude is given by \( wg(t^i_1) \) and it may be larger than this due to the presence of \( I_{N}^i(t^i_1) \), i.e. the OMAI that falls at the same time slot \( t^i_1 \). The output of the correlator is then passed through a matched filter and a sampler. The sampled values, taken at \( t = T_b \), are then passed through a threshold detector, which is set at the optimum level \( v = w-1 \). The PIM symbol stream will then be extracted from the interference, \( I_{N}^i(t^i_2) \). At detecting a pulse at the start of the \( k \)\(_{th}\) symbol, the PIM demodulator will ignores \((n-1)T_b\) time slots, and then start counting the number of empty slots there after until the next pulse from the \( k+1 \)\(_{th}\) symbol is detected (refer to Fig. 4a). The count number, which corresponds to the information, \( S^i_k \), is then converted into the desired \( M \) bit data sequence.
3.2. PROBABILITY OF BIT ERROR RATE

For the BER analysis, we have made the following assumptions: (i) each of $N$ users is asynchronously transmitting the $k^{th}$ symbol, (iii) the main source of interference is the signal received from the others users, $N - 1$, that are transmitting simultaneously, better known as the optical multiple access interference (OMAI), (iii) the transmitter and receiver is synchronised at chip (or slot) level $T_b$, (iv) the channel is ideal and non-dispersive and the normalized impulse response is a delta function $\delta(t)$, (v) noise due to the optical receiver and interference from fluorescent light are assumed to be negligible compared with the OMAI, and therefore are ignored, and (vi) the information $s^i_k$ is considered up to $2^M - 1 \leq n / 2$.

With reference to Fig. 4b, a false alarm pulse, due to OMAI, occurring within the $(n-1)T_b$ duration, which is previously occupied by the signature sequence, will result in no error being detected at all, one of the unique features of this scheme. This is because the demodulator on detecting the header pulse will ignore all the remaining $(n-1)T_b$ slots, as outlined in section 3. However, an error will occur only if a false alarm pulse falls within the information band $S^i_k$. The affect would be generation of two new symbols of shorter length for the $k^{th}$ symbol and longer length for the $k+1^{th}$ symbol, which the PIM demodulator will convert them back into two new data bits. In the example shown, the information at the output of CDMA decoder with no false alarm pulse will be demodulated as $S^i_k = 2$ and $S^i_{k+1} = 3$ (referred to the “correct demod” shown as [4]). With a false alarm pulse occurring in the $k^{th}$ symbol, the information will be demodulated as $S^i_k = 0$ and $S^i_{k+1} = 5$ (referred to the “wrong demod”). Hence, in hybrid PIM-CDMA system, an error in one symbol will affect two symbols, a characteristic, which does not exist in the hybrid PPM-CDMA due to its fixed frame structure. As seen from (5), at the CDMA
decoder, each receiving signal \( e_{\text{pim}}^r \) is correlated with its desired signature sequence. Since \((n, w, 1, 1)\) strict-OOC is designed such that other user signature has only one bit matched to the desired signature [11], then the probability of interference, also known as the cross-correlation constraints \( \lambda_c \), due to only one other user is given by:

\[
P_{\lambda_c-\text{pim}} = \frac{2w^2}{2n + L - 1}.
\]  

(6)

Whereas the probability of self-interference, also known as the auto-correlation constraints \( \lambda_a \), due to the target user \( e_{\text{pim}}^j \) is given by:

\[
P_{\lambda_a} = \frac{w(w-1)}{n-1}.
\]  

(7)

Equation 7 is true only for \( 2^M - 1 \leq n/2 \), is also equally applicable to hybrid PPM-CDMA. Therefore, there are two probabilities that false alarm pulse/s may occur due to (i) self-interference and OMAI, and (ii) OMAI only, as outlined below.

### 3.3. SELF-INTERFERENCE AND OMAI

The probability that a signal level due to both OMAI and the self-interference will exceed the threshold is given by:

\[
P_{\text{self}} = \gamma P_{\lambda_a} \sum_{v=1}^{N} \binom{N}{v} P_{\lambda_c}^{v-1} \left( 1 - P_{\lambda_c-\text{pim}} \right)^{N-v}
\]  

(8)

where the threshold level \( v_{\text{th}} = w-1 \), and \( \gamma \) is the number of information slots. In addition, it is possible that more than one false alarm pulse will occur in one or more frames, which may affect any two consecutive symbols. Since we have assumed that symbols transmitted are equal likely, then (8) may be written as:
$P_{self} = \frac{1}{2^{M-1}} P_{\lambda_a} \sum_{y=0}^{L-1} \frac{\gamma}{n+\gamma} \sum_{v=\nu_a}^{N} \binom{N}{v} P_{\lambda_k-pim}^{-1} \left(1 - P_{\lambda_k-pim}\right)^{N-v}.$ \hfill (9)

### 3.4. ONLY OMAI

The probability that a signal level, in the presence of OMAI, will exceed the threshold level is given by:

$$P_{OMAI} = \frac{1}{2^{M-1}} \left(1 - P_{\lambda_a}\right) \sum_{y=0}^{L-1} \frac{\gamma}{n+\gamma} \sum_{v=\nu_a}^{N-1} \binom{N-1}{v} P_{\lambda_k-pim}^{-1} \left(1 - P_{\lambda_k-pim}\right)^{N-1-v}.$$ \hfill (10)

The probability of false alarm error is just the summations of (9) and (10). For hybrid PIM-CDMA it is given by $P_{fe-pim} = P_{self} + P_{OMAI}$. Thus,

$$P_{fe-pim} = \frac{P_{\lambda_a}}{2^{M-1}} \sum_{y=0}^{L-1} \frac{\gamma}{n+\gamma} \left[ \sum_{v=\nu_a}^{N} \binom{N}{v} P_{\lambda_k-pim}^{-1} \left(1 - P_{\lambda_k-pim}\right)^{N-v} \right]$$

\begin{equation}
+ \left(1 - 1 \right) \sum_{v=\nu_a}^{N-1} \binom{N-1}{v} P_{\lambda_k-pim}^{-1} \left(1 - P_{\lambda_k-pim}\right)^{N-1-v}.
\end{equation}\hfill (11)

For comparison, the probability of false alarm for the hybrid PPM-CDMA is also given as:

$$P_{fe-ppm} = \frac{P_{\lambda_a}}{2^{M-1}} \sum_{y=0}^{L-1} \frac{\gamma}{n+L-1} \left[ \sum_{v=\nu_a}^{N} \binom{N}{v} P_{\lambda_k-ppm}^{-1} \left(1 - P_{\lambda_k-ppm}\right)^{N-v} \right]$$

\begin{equation}
+ \left(1 - 1 \right) \sum_{v=\nu_a}^{N-1} \binom{N-1}{v} P_{\lambda_k-ppm}^{-1} \left(1 - P_{\lambda_k-ppm}\right)^{N-1-v},
\end{equation}\hfill (12)

where

$$P_{\lambda_k-ppm} = \frac{W^2}{n+L-1}.$$ \hfill (13)

The probability of bit error rate is obtained using the following expression [19]:

$$P_b = \left(\frac{L}{2(L-1)}\right)P_{fe}.$$ \hfill (14)
Using (11) and (14), the BER for hybrid PIM-CDMA is calculated and the results versus number of user $N$ for different values of $w$, and $M = 5$ are shown in Fig. 5. In addition, for comparison the results are shown for hybrid PPM-CDMA and OOK-CDMA. All three schemes display improved BER performance as $w$ increases. For a given value of $w$, the BER for OOK-CDMA increases exponentially as $N$ increases. Whereas in hybrid PIM-CDMA and PPM-CDMA the BER increases as $N$ increases, reaching a peak value and then decreasing as $N$ increases further.

For hybrid PIM-CDMA, the maximum BER is recorded at $N = 30$ for $w = 9$, which are marginally lower than both the PPM and OOK. However, at a much larger values of $N$ it displays better performance than OOK. Fig. 8 shows the BER versus $N$ for different values of $M$. The BER performance improves as $M$ decreases. At $N > 20$ both PPM and PIM display better performance than OOK. For BER $= 10^{-9}$, the number users for PIM and PPM is less than 10, whereas for OOK it is about 12.

4. CONCLUSIONS

A hybrid PIM-CDMA modulation scheme for optical wireless communication systems has been presented. We have shown that the proposed scheme can increase the system transmission rate beyond the limit due to a fixed bit rate condition, and requires no frame synchronisation. Analysis for BER taking into account the effects of self-interference for both hybrid systems were presented and the results were compared with hybrid PPM-CDMA and OOK-CDMA. Hybrid PIM-CDMA displays a BER performance similar to hybrid PPM-CDMA and only marginally higher than OOK-CDMA at a low number of users. However, at high number users hybrid PIM-CDMA outperforms OOK-CDMA. Hybrid PIM-CDMA is an alternative candidate
for a fully asynchronous multiple access system which provides higher transmission rate, and low BER.

REFERENCES


List of figures

Fig. 1: Frame sequence for: (a) OOK, and (b) OOK-CDMA.

Fig. 2: Frame formats for $M = 2$ bits: (a) OOK-CDMA, (b) hybrid PPM-CDMA, and (c) hybrid PIM-CDMA.

Fig. 3: Hybrid PIM-CDMA system block diagram, (a) transmitter, and (b) receiver.

Fig. 4: Output of CDMA decoder for $k^{th}$ and $k+1^{th}$ symbols. (a) error free, and (b) with false alarm. [...] represents the demodulated data at output PIM demodulator.

Fig. 5: BER versus number user $N$ for various $w$ and $M=5$ bits.

Fig. 6: BER versus number user $N$ for various $M$ bits and $w=9$. 
Fig. 3
Fig. 4